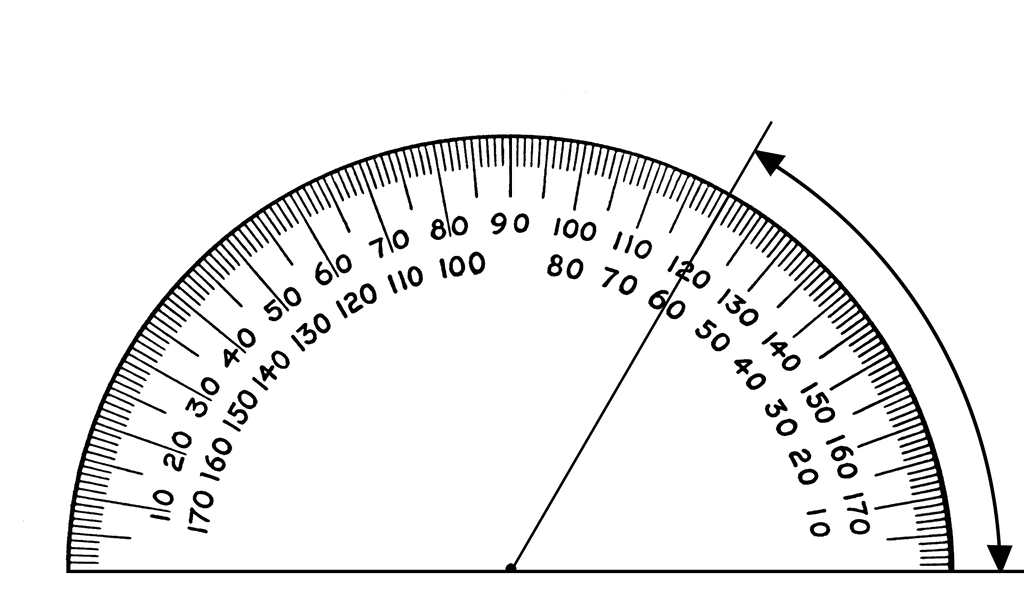
**Barron’s Math 360: A Complete Study Guide to Geometry**

# Chapter 2: Measure and Congruence

## Measurements of Segments and Angles

We use a *ruler* for measuring the length of a segment and the *protractor* for measuring an angle. In our example, the measure of the angle is 60 degrees. We abbreviate this by writing m∡, read as “The measure of angle is 60.” It is customary to omit the degree symbol (). Thus we never write m∡ or (omitting the “m”).



### Classifying Angles

Angles may be classified by comparing their measures to a angle. An L-shaped angle is called a ***right angle*** and its measure is exactly equal to . An angle whose measure is less than (but greater than ) is called an ***acute angle***. An angle whose measure is greater than (but les than ) is called an ***obtuse angle***. An angle that is exactly is called a **straight angle**.

A diagram of angles and angles

AI-generated content may be incorrect.

Note: It is customary to denote a right angle by marking a “box” in the corner of the angle as shown above.

## Betweenness of Points and Rays

Paul standing in line with Allan and Barbara

A black and blue line with a blue circle

AI-generated content may be incorrect.

Paul not standing in line with Allan and Barbara

A blue dot and black line

AI-generated content may be incorrect.

Paul is behind both Allan and Barbara

A black and white photo of a long thin line

AI-generated content may be incorrect.

### Definition of Betweenness

Point is between points if both of the following are met:

1. Points, are three different collinear points.

Condition 1 eliminates the Paul is not in line as a possibility, while Condition 2 eliminates the possibility of Paul is behind both Allan and Barbara.

Note: If m ∡AOP = 40 and m ∡POB = 10, then   
m ∡AOB = 50. This somewhat obvious relationship is given a special name: the Angle Addition Postulate.

A diagram of a triangle

AI-generated content may be incorrect.

**Angle Addition Postulate**

If ray lies in the interior of angle , then .

## Congruence

Figures that have the same size and shape are said to be congruent. The symbol for congruent is .

Figures may agree in one or more dimensions yet not be congruent. A square and parallelogram may have 4 sides of equal length, but the figures are not congruent if their corresponding angles are not identical in measure.

A line segment has a single dimension – its length. Two segments are congruent, therefore if they have the same length.

Similarly, if two angles have the same measure, then they are congruent.

Congruence is one of the fundamental concepts of geometry.

**Definition of Congruent Segments Or Angles**

Segments (or angles) are *congruent* if they have the same measure.

## Basic Constructions

Geometric constructions, unlike *drawings*, are made only with a straightedge (for example, an unmarked ruler) and compass. The point at which the pivot point of the compass is placed is sometimes referred to as the *center*, while the fixed compass setting that is used is called the radius length.

### Copying Segments And Angles

Given a line segment or angle, it is possible to construct another line segment or angle that is congruent to the original segment or angle without using a ruler or protractor.

### Midpoint and Bisector

### Definition of a Midpoint

### Definition of a Segment Bisector

### Definition of Angle Bisector

Diagrams and Drawing Conclusions

In general, we may assume only collinearity and betweenness of points. We may not make any assumptions regarding the measures of segments or angles unless they are given to us.

## Properties of Equality and Congruence

John is taller than Kevin and Kevin is taller than Louis. How do the heights of John and Louis compare?

We can analyze the situation with the aid of a simple diagram. This leads us to conclude that John must be taller than Louis.

**This Is The Key To The Method**

When directly comparing John with Louis, we have used a transitive property to conclude that John’s height is greater than Louis’ height. The “greater than” relation is an example of a relation that possesses the transitive property.

The equality (=) and congruence() relations possess the transitive property.

If angle is congruent to angle and angle is congruent to angle , then angle must be congruent to angle .

|  |  |  |
| --- | --- | --- |
| **Property** | **Equality Example** | **Congruence Example** |
| *Reflexive* |  |  |
| The identical expression may be written on either side of the symbol. Any property quantity is equal (congruent) to itself. |  |  |
| *Symmetric* |  |  |
| The positions of the expressions on either side of the symbol may be reversed. Quantities may be “flip-flopped” on either side of the sign. |  |  |
| *Transitive* |  |  |
| If two quantities are equal (congruent) to the same quantity, then they are equal (congruent) to each other. |  |  |

Another useful property of the equality relation is the *substitution* property. If , then an equivalent number may be substituted in place of the numerical expression on the right side of the equation. We may substitute 5 for , and write .

## Additional Properties of Equality

|  |  |  |
| --- | --- | --- |
| **Property** | **Algebraic Example** | **Formal Statement** |
|  |  |  |
| *Addition* (+) |  |  |
| The same (or =) quantities may be added to both sides of an equation. | If equals are added to: | Equals, there sums are equal  Or |
|  |  |  |
| *Subtraction* (-) |  |  |
| The same (or =) quantities may be subtracted from both sides of an equation. | If equals are subtracted: | Form equals, there differences are equal.  Or |
|  |  |  |
| *Multiplication* (x) |  |  |
| The same quantity may be used to multiply both sides of an equation | Solve for : | If equals are multiplied by equals, their products are equal.  Or |

The addition, subtraction and multiplication properties may be applied also to geometric situations.

## The Two-Column Proof Format

**This Is The Key To The Method**

A proof in geometry usually includes these four elements:

When doing a proof, it is important to draw a diagram or mark a given diagram with information and conclusions drawn from the given information so that you can plan out your line of reasoning.

Greek mathematicians wrote proofs in paragraph form. Most beginning geometry students, however, find it helpful to organize and record their thinking using a table like format.

## Summary

* Angles are classified by degree measure as
  + Right (90°)
  + Acute ()
  + Obtuse ()
  + Straight (180°)
* Figures are congruent () if and only if they agree in all dimensions.
* Midpoints and segment bisectors create two congruent segments.
* Angle bisectors create two congruent angles.
* Properties of equality (reflexive, symmetric, transitive, substitution, addition, multiplication, subtraction) can be used in the process of proving statements to be true.
* Precise geometric figures can sometimes be constructed using a compass and straightedge.

## Review Exercises

1. In the accompanying diagram, classify each of the following angles as acute, right, obtuse, or straight.
2. ∡TOM – acute
3. ∡LOM – straight
4. ∡SOM – right
5. ∡LOR – acute
6. ∡LOT – obtuse
7. ∡ROS – acute
8. ∡MOR – obtuse
9. Point P is between points and . If   
   .
10. If points are collinear and ,

and , which point is between the other two?

**is between points**



1. lies in the interior of angle ;

and . If , find the measure of the smallest angle formed.

1. bisects at point . If, find the length of RS.
2. bisects . If , find .

1. bisects . If and , classify angle as acute, right, or obtuse.

**Angle is 90, which is a right angle.**

1. If is the midpoint of , and   
   , find the length of .
2. In the accompanying diagram, pairs of angles and segments are indicated as congruent. Use the letters in the diagram to complete the following congruence relations.
3. GIVEN: bisects .

CONCLUSION:

1. GIVEN: bisects .

CONCLUSION:

1. GIVEN: bisects .

GIVEN: bisects .

CONCLUSION: ,

*In Exercises 13 to 16, justify the conclusion drawn by identifying the property used to draw the conclusion as reflexive, transitive, symmetric, or substitution.*

|  |  |  |
| --- | --- | --- |
| GIVEN: |  |  |
|  |  |  |
| CONCLUSION: |  | ***transitive*** |

|  |  |  |
| --- | --- | --- |
| GIVEN: | Quadralateral ABCD |  |
| CONCLUSION: |  | ***reflexive*** |

|  |  |  |
| --- | --- | --- |
| GIVEN: |  |  |
|  |  |  |
|  |  | Bisects definition |
| CONCLUSION: |  | ***transitive*** |

|  |  |  |
| --- | --- | --- |
| GIVEN: |  |  |
|  |  |  |
| CONCLUSION: |  | ***substitution*** |

*In exercises 17 to 23, indicate on the diagrams the corresponding pairs of equal or congruent parts. Then state the property of equality that can be used to justify each conclusion.*

|  |  |  |
| --- | --- | --- |
| GIVEN: |  |  |
|  |  | Identity |
|  |  | Addition |
| CONCLUSION: |  |  |
| Property: | ***Addition*** |  |

|  |  |
| --- | --- |
| GIVEN: |  |
|  |  |
|  |  |
|  |  |
|  | - |
| CONCLUSION: |  |
| PROPERTY: | ***Subtraction*** |

|  |  |
| --- | --- |
| GIVEN: |  |
|  |  |
|  |  |
| CONCLUSION: |  |
| PROPERTY: | ***Addition*** |

|  |  |  |
| --- | --- | --- |
|  | GIVEN: |  |
|  |  |  |
|  | REFLEXIVE: |  |
|  | REFLEXIVE: |  |
|  | SUBSTITUTION: |  |
|  | CONCLUSION: |  |
|  | PROPERTY: | ***Substitution*** |

|  |  |  |
| --- | --- | --- |
|  | GIVEN: |  |
|  | REFLEXIVE: |  |
|  | REFLEXIVE: |  |
|  | SUBSTITUTION: |  |
|  | SUBTRACTION: |  |
|  | CONCLUSION: |  |
|  | PROPERTY: | ***Subtraction*** |

|  |  |  |
| --- | --- | --- |
|  | GIVEN: |  |
|  | SUBTRACTION: |  |
|  | CONCLUSION: |  |
|  | PROPERTY: | ***Subtraction*** |

|  |  |  |
| --- | --- | --- |
| 23. | GIVEN: |  |
|  |  |  |
|  | ADDITION: |  |
|  | CONCLUSION: |  |
|  | PROPERTY: | ***Addition*** |

*In Exercises 24 to 27, supply the missing reasons.*

|  |  |  |  |
| --- | --- | --- | --- |
|  | PROOF: | Statements | Reasons |
|  |  |  |  |
|  |  | 3. JR = JR | Reflexive |
|  |  | 4. | Addition |

|  |  |  |
| --- | --- | --- |
|  | Statements | Reasons |
|  | 1. , | Given. |
|  | 2. | If two angles are congruent, they are equal in measure. |
|  | 3. | Subtraction property of equality. |
|  | 4. | Substitution property of equality. |
|  | 5. | If two angles are equal in measure, they are congruent. |

|  |  |  |
| --- | --- | --- |
|  | Statements | Reasons |
|  | PROOF OF PART A: |  |
|  | 1. | Given. |
|  | 2. | Reflexive. |
|  | 3. | Subtraction |
|  | 4. | Substitution |

|  |  |  |
| --- | --- | --- |
|  | PROOF OF PART B: |  |
|  | 5. | Given. |
|  | 6. | Congruent segments are equal in measure. |
|  | 7. | Reflexive |
|  | 8. | Subtraction. |
|  | 9. | Substitution. |
|  | 10. | Segments equal in measure are congruent. |

|  |  |  |
| --- | --- | --- |
|  | Statements | Reasons |
|  | 1. | Given. |
|  | 2. | Congruent angles are equal in measure. |
|  | 3. | Given. |
|  | 4. | Definition: Bisector divides an angle into two equal angles. |
|  | 5. | Symmetric. |
|  | 6. | Angles equal in measurement are congruent. |